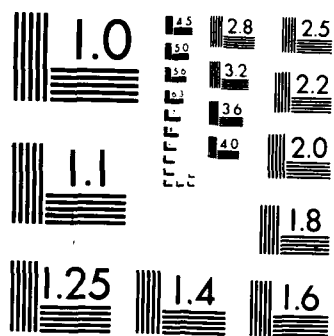


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T. M. Antonsen, Jr. and B. Levush

Plasma Preprint UMLPF #88-011

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OPTICAL GUIDING IN THE SEPARABLE BEAM LIMIT

T. M. Antonsen, Jr. and B. Levush
Laboratory for Plasma and Fusion Energy Studies
University of Maryland

ABSTRACT

The nonlinear theory of optical guiding in an FEL Amplifier is developed for the case in which the spatial dependence of the current source term in the wave equation can be separated into the product of a function of radius and a function of axial distance. Such a separation can be motivated if either the betatron wavelength is shorter than other lengths of interest (synchrotron wavelength, vacuum Rayleigh length) or if the radiation waist exceeds the beam radius. In this limit with the choice of a Gaussian profile for the electron beam density the wave equation can be solved exactly and the radiation field felt by the particles can be expressed as a one-dimensional convolution of the current source.

With the given expression for the radiation field, the equations of motion can be solved in the trapped particle regime. Requiring consistency between the particle motion and the fields yields expressions describing nonlinear guided states. The adiabatic evolution of these guided states in the presence of a tapered wiggler is determined by conservation of the electrons' action and total (field + electron beam) energy. Using these relations the growth of the radiation waist as the beam is decelerated can be calculated.

I. INTRODUCTION

Optical guiding in free electron lasers can become a complicated numerical problem in the nonlinear regime due to the three dimensional structure of the wave fields.¹⁻⁸ Analytic treatments for the linear regime of exponential gain have been developed.²⁻⁵ For nonlinear simulations the representation of the field can be accomplished either by finite difference methods^{3,6} or expansions in a set of basis functions.⁷ Invariably questions are raised concerning the accuracy of these representations. The purpose of this paper is to show that with a few simple heuristic approximations to the current source term in the wave

equation the wave equation can be solved exactly. The result is that the particle motion can be treated as in a one dimensional model and many analytic results describing the effects of diffraction can be obtained. Furthermore, this model, whose solution can be obtained analytically, can serve as a test bed for the other methods. The model we present is similar to one suggested by Moore in the strong diffraction limit.⁴

We begin with a consideration of the wave equation for the signal vector potential A_s which is appropriate for amplifiers,

$$\begin{aligned} \tilde{A} &\approx \hat{e}_y (A_s e^{i\omega(z/c-t)} + c.c) \\ \nabla_{\perp}^2 A_s(x_{\perp}, z) + 2i \frac{\omega}{c} \frac{\partial A_s}{\partial z} &= \frac{4\pi}{c} \delta j(x_{\perp}, z) \end{aligned} \quad (1)$$

where $\delta j(x_{\perp}, z)$ gives the complex amplitude of the component of the beam current density oscillating in phase with the signal wave. The current density will be localized in the transverse dimension to an area the size of the electron beam. Furthermore, we can expect if all the electrons are interacting strongly with the signal field the z dependence of δj at different radial positions x_{\perp} will be strongly correlated. Accordingly, we will make the approximation that the axial and transverse dependence of δj can be separated,

$$\delta j = \frac{v_y I}{\pi r_b^2 v_z} \exp\left(-\frac{r^2}{2r_b^2}\right) s(z) \quad (2)$$

where

$$v_y = \frac{q A_w}{mc \gamma_r}$$

is the jitter velocity induced by the wiggler field, v_z and I are the speed and current of the electron beam, and we have taken the electron beam to have a Gaussian profile in radius characterized by a width r_b . Here we have assumed a planar wiggler. In Eq. (2) the function $s(z)$ gives the axial dependence of the current density and will be determined by the dynamics of the interaction of the beam particles with the ponderomotive wave. By analogy with the one dimensional problem we write

$$s(z) = \langle e^{-i\psi(z)} \rangle \quad (3)$$

where $\psi(z)$ represents the phase of a particle in the beat wave potential, and the angular average is over initial particle phases.

This separation is probably not justifiable rigorously except when the radiation waist is much larger than the electron beam radius and details of the radial dependence of the current source are washed out. This is essentially the argument advanced by Moore.⁴

The particles in our model evolve according to the standard one-dimensional pendulum equation,⁸

$$\frac{d}{dz} (\delta\gamma + \gamma_R) = - \frac{\partial}{\partial \psi} \left(\frac{\omega v_y}{v_z c} \right) \frac{1}{2} \left[\frac{q \bar{A}_s}{mc^2} e^{i\psi} + \frac{q \bar{A}_s^*}{mc^2} e^{-i\psi} \right] \quad (4a)$$

$$\frac{d}{dz} \psi = - \delta\gamma \frac{\partial}{\partial \gamma_R} \left(\frac{\omega}{v_z (\gamma_R, z)} \right) \quad (4b)$$

where $\gamma_R(z)$ is the so called resonant value of γ that is required to keep particles in phase with beat wave in the presence of tapered wiggler parameters, and $\delta\gamma$ is the deviation of a particle's actual γ from the resonant value. The quantity \bar{A}_s is the effective field that a beam particle feels and represents some radial average of the actual field $A_s(r, z)$. In our separable beam approximation there is no "first principles" prescription for making this average. However, it can be verified afterward that if one makes the choice

$$\bar{A}_s(z) = \int_0^\infty \frac{2\pi r dr}{\pi r_b^2} \exp\left(-\frac{r^2}{r_b^2}\right) A_s(r, z) \quad (5)$$

then a conservation of energy relation exists for the system. Accordingly, we pick Eq. (5) to define $\bar{A}_s(z)$.

The system of equations (4) can be put into the dimensionless form

$$\frac{\partial \psi}{\partial \xi} = p, \quad (6a)$$

$$\frac{\partial p}{\partial \xi} = - \frac{\partial}{\partial \psi} \frac{1}{2} \left(\bar{a} e^{i\psi} + \bar{a}^* e^{-i\psi} \right) - \alpha, \quad (6b)$$

where the normalizations of $\delta\psi$, and \bar{a} are given in Ref. 9 with L replaced by the Rayleigh length $L_R = r_b^2 \omega / c$ and

$$\alpha = \frac{\omega L_R^2}{c (\gamma_R^2 - 1)^{3/2}} \frac{\partial \gamma_R}{\partial z} \quad (7)$$

measures the deceleration due to tapering. The wave field satisfies the Schrodinger like equation,

$$\nabla_\perp^2 a + 2i \frac{\partial}{\partial \xi} a = \frac{\hat{I}}{\pi} \exp(-\rho^2) s(\xi) \quad (8)$$

where

$$\hat{I} = \frac{4\pi I}{(\gamma_R^2(0) - 1)^{3/2} I_A} \left(\frac{\omega L_R v_y(0)}{c v_z(0)} \right)^2 \quad (9)$$

measures the current and $I_A = mc^3/q = 1.7 \times 10^4$ Amps is related to the Alfvén current, and $\rho = r/r_0$ is the dimensionless radius. The only other parameters entering the system are the dimensionless beam current \hat{I} , and the distribution of injected momenta $f(p, \xi=0)$.

From this system of equations we may derive the following energy conservation law

$$\frac{\partial}{\partial \xi} \left[\hat{I} \langle p \rangle + \int_0^\infty 2\pi \rho d\rho |a|^2 \right] = -\alpha \hat{I} \quad (10)$$

II. SOLUTION OF THE WAVE EQUATION

We now turn to the wave equation (8). Because the radial dependence of the source is prescribed to be Gaussian we can solve for the wave field in terms of a Green's function $G(\rho, \xi - \xi')$ which satisfies

$$\nabla_\perp^2 G + 2i \frac{\partial G}{\partial \xi} = \frac{\exp(-\rho^2)}{\pi} \delta(\xi - \xi') \quad (11)$$

The resulting expression for the wave field is then expressed as a convolution,

$$a(\rho, \xi) = \hat{I} \int_0^\xi d\xi' G(\rho, \xi - \xi') S(\xi') + a_h(\rho, \xi) \quad (12)$$

where $a_h(\rho, \xi)$ represents the homogeneous vacuum solution corresponding to the initial (injected) wave field. Equation (11) is easily solved to yield,

$$G(\rho, \xi - \xi') = \frac{1}{2\pi i} \frac{\exp(-\rho^2/(1 + 2i(\xi - \xi')))}{(1 + 2i(\xi - \xi'))} \quad (13)$$

and the radial average in Eq. (5) can be performed to obtain $\bar{a}(\xi)$

$$\bar{a}(\xi) = \frac{\hat{I}}{4\pi i} \int_0^\xi d\xi' \frac{S(\xi')}{1 + i(\xi - \xi')} + \bar{a}_h(\xi) \quad (14)$$

Thus, we see that the effective field felt by the particles can be written as a one-dimensional convolution of the current source.

The kernel, $(1 + i(\xi - \xi'))^{-1}$, has an obvious physical interpretation. For small values of $(\xi - \xi')$ the wave field is just an integral of source. This is the result that would be obtained if we had dropped the transverse derivatives in the wave equation and represents the fact that nearby contributions of the source to the field are the same as if there

were no diffraction. For large values of $\xi - \xi'$ (recall distance is normalized to the Rayleigh length) the Kernel decays weakly indicating the fact that distant contributions of the source to field are reduced by diffraction. We note that this decay is very weak and that replacing the transverse derivatives in Eq. (7) by a constant loss rate which yield an exponential decay of the kernel greatly overestimates the decay.

It is interesting to examine the amplitude of the field that is predicted when the source has an exponential dependence on axial position, $S(\xi) = S_0 e^{i\nu\xi}$. Such a dependence certainly occurs in linear theory. We shall see also that this dependence occurs for steady nonlinear states as well. It is found that as $\xi \rightarrow \infty$

$$\bar{a}(\xi) = a_0 e^{i\nu\xi}$$

with

$$a_0 = \hat{I} S_0 E(\nu), \quad (15)$$

and

$$E(\nu) = \frac{1}{4\pi i} \int_0^\infty \frac{dx e^{-i\nu x}}{1 + ix}. \quad (16a)$$

The complex quantity $E(\nu)$ is related to the exponential integral

$$E(\nu) = -\frac{1}{4\pi} e^\nu E_1(\nu), \quad (16b)$$

with

$$E_1(\nu) = \int_\nu^\infty \frac{dt e^{-t}}{t}.$$

Using Eq. (6b) we can calculate the rate at which energy is extracted from the particles to maintain this field,

$$\begin{aligned} \frac{\partial}{\partial \xi} \hat{I} \langle p \rangle &= -\frac{i\hat{I}}{2} [\bar{a} S^* - \bar{a}^* S] \\ &= -i \frac{\hat{I}^2}{2} |S_0|^2 e^{i(\nu - \nu^*)\xi} [E(\nu) - (E(\nu))^*]. \end{aligned} \quad (17)$$

Thus, the imaginary part of $E(\nu)$ gives the rate at which energy is extracted from the particles. For ν real and positive $E(\nu)$ is also real and the field is maintained in steady state without energy being supplied by the particles. If ν is real and negative we can manipulate the contour in Eq. (16) to show

$$E(\nu) = \frac{P}{4\pi} \int_0^\infty \frac{dy e^{-|\nu|y}}{1-y} - \frac{i}{4} e^{-|\nu|}$$

where P indicates the principal part of the integral. The particles lose energy according to

$$\frac{\partial}{\partial \xi} \hat{I} \langle p \rangle = -\frac{\hat{I}^2}{4} |S_0|^2 e^{-|\nu|}.$$

This energy is radiated away from the beam and represents a situation in which optical guiding is not present.

The preceding discussion shows that the key requirement for optical guiding in our model is that $v > 0$.^{1,8} Here our definition of an optically guided state is one that can be maintained without the continuous transfer of energy from the particles to the fields. Physically, the condition $v > 0$ is simply the requirement that the axial phase speed of the perturbed current source be less than the speed of light. In this case the fields outside the electron beam are evanescent and field energy is confined to the vicinity of the electron beam.

Equation (17) may also be used to calculate the field energy stored in an optically guided state with $v > 0$. We imagine such a state is created adiabatically and let $v = v_r + iv_i$ with $v_i < 0$ and $|v_r| \gg |v_i|$. We find by using Eqs. (9) and (17)

$$\frac{\partial}{\partial \xi} \int_0^{\infty} 2\pi \rho d\rho |a|^2 = - |\hat{IS}_0|^2 e^{-2v_i \xi} v_i \frac{\partial}{\partial v_r} E(v_r) .$$

Integrating from $\xi = -\infty$ to ξ and letting v_i approach zero we have

$$\int_0^{\infty} 2\pi \rho d\rho |a|^2 = \frac{1}{2} \frac{\partial E / \partial v_r}{E^2} |a_0|^2 . \quad (18)$$

Thus, we can now express the field energy in a guided state in terms of the amplitude of the effective field a_0 .

Finally, we define an expression for the beam to radiation filling factor, F .

$$F = \left| \int_0^{\infty} 2\pi \rho d\rho \frac{a}{\pi} \exp(-\rho^2) \right|^2 / \int_0^{\infty} 2\pi \rho d\rho |a|^2 \quad (19a)$$

or for steady states,

$$F = 2E^2(v) / (\partial E / \partial v) . \quad (19b)$$

Using Schwartz's inequality one can show from Eq. (19a) $F < (2\pi)^{-1}$. The limit $F = (2\pi)^{-1}$ is achieved in steady state for large v which follows from the asymptotic expansion of Eq. (16). This conforms with our interpretation of the kernel. For large v contributions to the source from nearby positions dominate. As a result, diffraction is not important, and the beam and radiation are perfectly matched. On the other hand, for small v we obtain $F \sim (2\pi)^{-1} v \ln^2 v$. In this case, due to diffraction the electron beam occupies a much smaller area than the radiation field. Note, however, that F decreases slowly with v . For $v = .01$ we find

$2\pi F \approx 0.27$. This points out that diffraction is a weak effect.

III. LINEAR THEORY

In this section we will analyze the linear regime described by our separable beam model. To aid in this we augment the particle equations with the corresponding Vlasov equation

$$\frac{\partial f}{\partial \xi} + p \frac{\partial f}{\partial \psi} - \frac{1}{2} (a e^{i\psi} - a^* e^{-i\psi}) \frac{\partial f}{\partial p} = 0 \quad (20)$$

where $f(p, \psi, \xi)$ is the distribution function. In equilibrium, $f = f_0(p)$. Linearizing Eq. (20), assuming exponential dependences of the first order quantities on ξ , and using Eq. (15) to relate the field amplitude to the perturbed distribution function yields the dispersion relation for v ,

$$1 = \frac{\hat{I} E(v)}{2} \int \frac{dp}{(v+p)} \frac{\partial f_0(p)}{\partial p} . \quad (21)$$

Here, $f_0(p)$ is normalized to unity. For a cold beam ($f_0 = \delta(p-p_0)$) we can perform the integral in Eq. (21) to obtain $1 = [\hat{I} E(v)]/[2(v+p_0)^2]$. For large beam currents $\hat{I} \gg 1$ we anticipate v will be large in which case we can use the large argument limit of $E(v)$ to obtain $v(v+p_0)^2 = -\hat{I}/8\pi$, which is identical to the corresponding one-dimensional dispersion relation. Thus, if \hat{I} is large the effects of diffraction are negligible mainly because fields exponentiate in distance shorter than the Rayleigh length. For a perfectly matched beam ($p_0 = 0$) the spatial growth rate is found to have the characteristic third root dependence on current $v_i = (-\sqrt{3/2})(\hat{I}/8\pi)^{1/3}$. If \hat{I} is small, and the beam energy is perfectly matched we expand $E(v)$ for small v to obtain $v^2 = (-\hat{I}/8\pi) \ln v^{-1}$, which yields

$$v_i \sim - \left(\frac{\hat{I}}{8\pi} \right)^{1/2} \ln^{1/2} \left[\left(\frac{\hat{I}}{8\pi} \right)^{-1/2} \right] .$$

In this case the growth rate has a slightly weaker dependence on current (between $\hat{I}^{1/2}$ and $\hat{I}^{1/3}$) owing to diffraction.

A detailed analysis of the dispersion relation, Eq. (21) including the effects of beam energy mismatch and energy spread will be presented in a future publication.

IV. NONLINEAR GUIDED STATES

We now consider cases where the field amplitude is large enough to trap particles nonlinearly in the beat wave potential. Further, we assume that particles execute many synchrotron oscillations before the parameters of the beat wave change. To treat this case analytically we let the field be expressed in terms of a slowly varying amplitude and rapidly varying phase $\bar{a}(\xi) = a_0 \exp(i \int^\xi v(\xi') d\xi')$ where $v(\xi)$ is anticipated to be real and positive, and varies slowly with ξ in response to changes in the wiggler parameters. In the particle equations we define a new phase $\hat{\psi} = \psi + \int^\xi v(\xi') d\xi'$ which results in the pair of equations

$$\frac{\partial \hat{\psi}}{\partial \xi} = p + v, \quad \frac{\partial p}{\partial \xi} = - \frac{\partial}{\partial \hat{\psi}} a_0 \cos \hat{\psi}, \quad (22)$$

where we have taken a_0 to be real and we have neglected the deceleration α to lowest order. Equations (22) are derivable from the Hamiltonian H

$$H = \frac{p^2}{2} + pv + a_0 \cos \hat{\psi}. \quad (23)$$

Thus, in the trapped particle regime the distribution function depends only on H , $f = f(H)$. We can now evaluate the source term in Eq. (14) and obtain the expression for the value of the wave amplitude a_0 . In doing this we assume that $v(\xi)$ and all other slowly varying quantities are constant on a length scale in ξ of order unity. (In fact, for small v , because of the slow decay of kernel we must make the more stringent assumption that these quantities are constant on a scale of order v^{-1} .) We then perform the convolution integral in Eq. (14) as if the source had a simple $\exp(iv\xi)$ dependence. Converting the p integration in Eq. (24) to an H integration, we obtain

$$a_0 = \hat{I}E(v) \int \frac{d\hat{\psi} dH f(H) \cos \hat{\psi}}{\pi \left[2\left(H - \frac{v^2}{2} - a_0 \cos \hat{\psi}\right) \right]^{1/2}}. \quad (24)$$

For positive values of a_0 it can be seen from the Hamiltonian that deeply trapped particles will have $\cos \hat{\psi} < 0$. Thus, if optically guided states exist, the average

$$\langle \cos \hat{\psi} \rangle = \int \frac{d\hat{\psi} dH f(H) \cos \hat{\psi}}{\pi \left[2\left(H - \frac{v^2}{2} - a_0 \cos \hat{\psi}\right) \right]^{1/2}},$$

must be negative. Examining $E(v)$ given by Eq. (16a) or (16b) it is seen

that for real positive v $E(v)$ is negative and thus Eq. (24) is consistent for positive a_0 and steady guided states exist. The existence of these guided states has nothing to do with the radial profile of the beam, rather it is a consequence of the saturated states predicted by the one-dimensional model. The crucial requirement is that in a saturated state the phase speed of the current perturbation is below the speed of light so that energy can not be radiated laterally from the beam.

Equation (24) indicates that even for very weak beams $\hat{I} \ll 1$ guided states exist. This is due to the logarithmic divergence of $E(v)$ for small v . However, the filling factor for these states will be very small so there utility may be limited.

The slow evolution of the guided states described by Eq. (24) can be determined by returning to the governing equations, and restoring the deceleration α . The result is that f is a function of H through the conserved particle action $J = \int d\psi p$. The parameter v is then determined from the energy conservation relation

$$\hat{I} \langle p \rangle + \frac{1}{2} a_0^2 \frac{\partial E / \partial v}{E^2(v)} = - \hat{I} \int_0^\xi d\xi \alpha(\xi)$$

where the average of $\langle p \rangle$ is found to be simply $-v$. Thus, using Eq. (24) we can write the energy balance equation,

$$\frac{\hat{I}}{2} \frac{\partial E}{\partial v} \langle \cos \psi \rangle^2 - v = - \int_0^\xi d\xi \alpha$$

Recalling the definition of E we see that $\partial E / \partial v$ is positive and decreases with v . Thus, assuming the dependence of $\langle \cos \psi \rangle^2$ on v to be weak, we see that for acceleration ($\alpha > 0$) $v(\xi)$ increases with ξ corresponding to improved guiding (i.e., a larger filling factor). For deceleration, ($\alpha < 0$) $v(\xi)$ decreases with ξ corresponding to a decrease in the filling factor. These results must be regarded as somewhat qualitative because of the assumption of adiabatic change in the wiggler parameters. A more quantitative evaluation awaits the numerical simulation of the governing equations.

V. CONCLUSIONS

In this paper we have presented a model system of equations (the separable beam model) which is capable of describing a wide variety of phenomena associated with optical guiding in free electron lasers. The

model's virtues are that it allows for an exact treatment of the wave fields and collapses the description of the FEL interaction back to a slightly modified version of the much studied one-dimensional model.

Unfortunately, the key step in obtaining this model is not derivable from first principles. That is the separation of variables in the current source term is not rigorously derived. We argue however, that the details of the radial distribution of current are not important for beams with small or moderate filling factors. These are beams with insufficient current to produce exponentiation of fields in one Rayleigh length. In some respect optical guiding in these beams is of most interest since optical guiding in strong beams is more assured.

Finally, we suggest that due to the exact solvability of the radiation field in the separable beam model, that it provides a test by which more sophisticated treatments can be judged.

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